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On Soft Semi-Open Sets

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Abstract— The objective of this paper is to describe the basics of soft semi-open sets and soft semi-closed sets in soft topological spaces by applying the functions of D. Molodstov's soft set theory.

Keywords— soft set, soft topology, soft open sets, soft closed sets, soft semi-open set and soft semi-closed set.

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1 INTRODUCTION

MANY researchers followed Molodtsov [4] when he introduced soft set theory as a basic mathematical application in describing with the ambiguity of not clearly defined objects. The practical problems in engineering, social science, life science were solved using these mathematical applications. The soft topological spaces and its basic notations were dealt in detail by Shabir and Naz [10].

The equality of two soft sets, subset and superset of Soft set, complement of a soft set, null soft set and absolute soft set with examples were explained by Maji [7]. Current researchers are now dealing with the latest advanced techniques by applying the results in operations research, Riemans integration, Game theory, theory of probability and arrived at the results for some basic notations of soft set theory. The foundations of the theory of soft topological spaces to open the door for experiments for soft mathematical concepts and structures that are based on soft set-theoretic operations were dealt by Naim et.al., [8]. In the recent research papers, in depth study on Soft topology were studied in the papers [1 , 2 , 3 , 5 , 6 , 9 , 11 , 12]. A new soft set called soft semi-open set and soft semi-closed sets in soft topological space were studied theoretically in this paper.

2. PRELIMINARIES:

The following definitions are essential for the development of the paper.

Definition 2.1. [4] Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U. The pair (F,E) or simply F_E , is called a soft set over U, where F is a mapping given by $F: E \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in U$, $F(e)$ may be considered as the set of e-approximate elements of the soft set F . The collection of all soft sets over U and E is denoted by $S(U)$. If $A \subseteq E$, then the pair (F,A) or simply F_A, is called a soft set over U, where F is a mapping $F: A \to P(U)$. Note that for $e \notin A$, $F(e) = \phi$.

Definition 2.2. [11] The union of two soft sets of F_B and G_C over the common universe U, is the soft set H_D, where B and C are subsets of the parameter set E, $D = B \cup C$ and for all $e \in D$, $H(e) = F(e)$ if $e \in B - C$, $H(e) = G(e)$ if $e \in C - B$ and $H(e) = F(e) \cup G(e)$ if $e \in B \cap C$, we write $F_B \tilde{\cup} G_C = H_D$.

Definition 2.3. [11] The intersection of two soft sets of F_B and G_C over the common universe U is the soft set H_D , where $D = B \cap C$ and for all $e \in D$, $H(e) = F(e) \cap G(e)$ if $D = B \cap C$. We write $F_B \tilde{\cap} G_C = H_D$.

Definition 2.4. [11] Let F_B and G_C be soft sets over a common universe set U and B, C □ E . Then F_B is a soft subset of G_C, denoted by $F_B \tilde{\subseteq} G_C$, if (i) B□ C and (ii) for all $e \in B$, $F(e) = G(e)$. Also G_C, is called the soft super set of F_B and is denoted by $F_B \tilde{\supseteq} G_C$.

Definition 2.5. [11] The soft sets F_B and G_C over a common universe set U are said to be soft equal, if $F_B \tilde{\subseteq} G_C$, and $F_B \tilde{\supseteq} G_C$. Then we write $F_B = G_C$.

Definition 2.6. [11] A soft set F_B over U is called a *null soft set* denoted by F_ϕ, if for all $e \in B$, $F(e) = \phi$.

Definition 2.7. [10] The relative complement of a soft set F_A , denoted by F_A^c, is defined by the approximate function

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$f_{A^c}(e) = f_A^c(e)$, where $f_A^c(e)$ is the complement of the set $f_A(e)$, that is $f_A^c(e) = U - f_A(e)$ for all $e \in E$. It is easy to see that $(F_A^c)^c = F_A$, $F_\phi^c = F_E$ and $F_E^c = F_\phi$.

Definition 2.8. [11] Let U be an initial universe and E be a set of parameters. If $B \subseteq E$, the soft set F_B over U is called an absolute soft set, if for all $e \in B$, F (e) = U.

The following is the definition of soft topology used by various authors.

Definition 2.9. [1] Let U be an initial universe and E be a set of parameters. Let τ be a sub collection of S(U), the collection of soft sets defined on U. Then τ is a soft topology if it satisfies the following conditions.

(i) $F_\phi, F_E \in \tau$.
(ii) The union of any number of soft sets in τ belongs to τ.
(iii) The intersection of any two soft sets in τ belongs to τ.

If this definition is considered for further development, then the results are similar to that of results in topological spaces. Therefore, throughout the paper the following definition of soft topology is used.

Definition 2.10. [10] Let U be an initial universe and E be a set of parameters. Let $F_A \in S(U)$. A soft topology on F_A, denoted by $\tilde{\tau}$, is a collection of soft subsets of F_A having the following properties:

1. F_ϕ, $F_A \in \tilde{\tau}$.
2. $\{F_{A_i} \tilde{\subseteq} F_A : i \in I\} \tilde{\subseteq} \tilde{\tau} \Rightarrow \tilde{\cup}_{i \in I} F_{A_i} \in \tilde{\tau}$.
3. $\{F_{A_i} \tilde{\subseteq} F_A : 1 \le i \le n,\ n \in \mathbb{N}\} \tilde{\subseteq} \tilde{\tau} \Rightarrow \tilde{\cap}_{i=1}^{n} F_{A_i} \in \tilde{\tau}$.

The pair $(F_A, \tilde{\tau})$ is called a soft topological spaces.

Definition 2.11. [9] Let $(F_A, \tilde{\tau})$ be a soft topological space in F_A. Elements of $\tilde{\tau}$ are called soft open sets. A soft set F_B in F_A is said to be a soft closed set in F_A, if its relative complement F_B^c belongs to $\tilde{\tau}$.

Definition 2.12. [12] Let $(F_A, \tilde{\tau})$ be a soft topological space and F_B be a soft set in F_A.
(i) The soft interior of F_B is the soft set $int(F_B) = \tilde{\cup}\{F_C : F_C$ is soft open and $F_C \tilde{\subseteq} F_B\}$.
(ii) The soft closure of F_B is the soft set $cl(F_B) = \tilde{\cap}\{F_C : F_C$ is soft closed and $F_B \tilde{\subseteq} F_C\}$.

Lemma 2.13. [12] Let $(F_A, \tilde{\tau})$ be a soft topological space and F_B and F_C be a soft set in F_A. Then the following hold.
(i) $int(int(F_B)) = int(F_B)$.
(ii) $F_B \tilde{\subseteq} F_C$ implies $int(F_B) \tilde{\subseteq} int(F_C)$.
(iii) $int(F_B) \tilde{\cap} int(F_C) = int(F_B \tilde{\cap} F_C)$.
(iv) $int(F_B) \tilde{\cup} int(F_C) \tilde{\subseteq} int(F_B \tilde{\cup} F_C)$.

Lemma 2.14. [12] Let $(F_A, \tilde{\tau})$ be a soft topological space and F_B and F_C be a soft set in F_A. Then the following hold.
(i) $cl(cl(F_B)) = cl(F_B)$.
(ii) $F_B \tilde{\subseteq} F_C$ implies $cl(F_B) \tilde{\subseteq} cl(F_C)$.
(iii) $cl(F_B) \tilde{\cap} cl(F_C) \tilde{\supseteq} cl(F_B \tilde{\cap} F_C)$.
(iv) $cl(F_B) \tilde{\cup} cl(F_C) = cl(F_B \tilde{\cup} F_C)$.

Proposition 2.15. [2] Let $(F_A, \tilde{\tau})$ be a soft topological space over F_A. Then
(i) F_ϕ, F_E are soft closed sets in F_A.
(ii) The union of any two soft closed sets is a soft closed set in F_A.
(iii) The intersection of any family of soft closed sets is a soft closed set in F_A.

3. Soft semi-open sets and Soft semi-closed sets:

This section is devoted to the study of soft semi-open sets and soft semi-closed sets.

Definition 3.1. Let F_B be a soft subset of a soft topological space $(F_A, \tilde{\tau})$. F_B is said to be a soft semi-open set, if $F_B \tilde{\subseteq} cl(int(F_B))$.

Example 3.2.
Let $U = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2\} \tilde{\subseteq} E$.
$F_A = \{(e_1, \{a, b, c\}), (e_2, \{a, b, c\})\}$,
$F_1 = \{(e_1, \{a\}), (e_2, \{b, c\})\}$, $F_2 = \{(e_1, \{b\}), (e_2, \{a, c\})\}$,
$F_3 = \{(e_2, \{c\})\}$, $F_4 = \{(e_1, \{a, b\}), (e_2, \{a, b, c\})\}$,
$F_5 = \{(e_1, \{a, c\}), (e_2, \{b\})\}$, $F_6 = \{(e_1, \{a\}), (e_2, \{b\})\}$,
$F_7 = \{(e_1, \{a, c\}), (e_2, \{b, c\})\}$, $F_8 = \{(e_1, \{b, c\}), (e_2, \{a, b, c\})\}$,
$F_9 = \{(e_1, \{c\}), (e_2, \{a, b\})\}$, $F_{10} = \{(e_1, \{b, c\}), (e_2, \{a\})\}$,
$F_{11} = \{(e_1, \{a, c\}) (e_2, \{b\})\}$, $F_{12} = \{(e_1, \{a, b, c\}) (e_2, \{a, b\})\}$,
$F_{13} = \{(e_1, \{c\})\}$, $F_{14} = \{(e_1, \{b\}), (e_2, \{a, c\})\}$,
$F_{15} = \{(e_1, \{b, c\}), (e_2, \{a, c\})\}$, $F_{16} = \{(e_1, \{b\}), (e_2, \{a\})\}$,
$F_{17} = \{(e_1, \{a\})\}$, $F_{18} = \{(e_1, \{a, b\}), (e_2, \{c\})\}$ $F_{19} = F_A$, $F_{20} = F_\phi$
$\tilde{\tau} = \{F_\phi, F_A, F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9\}$. Then $(F_A, \tilde{\tau})$ is a soft topological space. The family of all soft closed sets is $\{F_A, F_\phi, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}, F_{17}, F_{18}\}$. The family of soft semi-open set is $\{F_\phi, F_A, F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{11}, F_{12}, F_{14}, F_{15}, F_{18}\}$. The family of soft semi-closed set is $\{F_A, F_\phi, F_2, F_3, F_5, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}, F_{17}, F_{18}\}$.

Theorem 3.3. Every soft open set in a soft topological space

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$(F_A, \tilde{\tau})$ is a soft semi-open set.

Proof. The proof follows from the Definition 3.1 □

The following Example 3.4 shows that the reverse implication of Theorem 3.3 is not true.

Example 3.4. Consider the soft topological space of Example 3.2. Here F_{11}, F_{12}, F_{14}, F_{15} and F_{18} are soft semi-open sets but not soft open sets, since F_{11}, F_{12}, F_{14}, F_{15}, $F_{18} \notin \tilde{\tau}$.

Remark: F_ϕ and F_A are always soft semi-closed sets and soft semi-open sets.

Proposition 3.5. A soft set F_B in a soft topological space $(F_A, \tilde{\tau})$ is a soft semi-open set if and only if there exists a soft open set F_C such that $F_C \tilde{\subseteq} F_B \tilde{\subseteq} cl(F_C)$.

Proof. Assume that $F_B \tilde{\subseteq} cl(int(F_B))$. Then for $F_C = int(F_B)$, we have $F_C \tilde{\subseteq} F_B \tilde{\subseteq} cl(F_C)$. Therefore, the condition holds. Conversely, suppose that $F_C \tilde{\subseteq} F_B \tilde{\subseteq} cl(F_C)$ for some soft open set F_C. Since $F_C \tilde{\subseteq} int(F_B)$ and so $cl(F_C) \tilde{\subseteq} cl(int(F_B))$. Hence $F_B \tilde{\subseteq} cl(F_C) \tilde{\subseteq} cl(int(F_B))$. Hence F_B is soft semi-open set. □

Theorem 3.7. Let $(F_A, \tilde{\tau})$ be a soft topological space and $\{(F_{B_\alpha}) : \alpha \in \Delta\}$ be a collection of soft semi-open sets in $(F_A, \tilde{\tau})$. Then $\cup_{\alpha\in\Delta} F_{B_\alpha}$ is also a soft semi-open set.

Proof. Let $\{(F_{B_\alpha}) : \alpha \in \Delta\}$ be a collection of soft semi-open set in $(F_A, \tilde{\tau})$. Then for each $\alpha \in \Delta$, we have a soft open set $F_{C_\alpha} \tilde{\subseteq} F_{B_\alpha}$ such that $F_{C_\alpha} \tilde{\subseteq} F_{B_\alpha} \tilde{\subseteq} cl(F_{C_\alpha})$. $\cup_{\alpha\in\Delta} F_{C_\alpha} \tilde{\subseteq} F_{B_\alpha} \tilde{\subseteq} \cup_{\alpha\in\Delta} cl(F_{C_\alpha}) \tilde{\subseteq} cl(\cup_{\alpha\in\Delta} F_{C_\alpha})$. □

Definition 3.8. A soft set F_B in a soft topological space $(F_A, \tilde{\tau})$ is said to be a soft semi-closed set, if its relative complement is a soft semi-open set.

Theorem 3.9. Every soft closed set in a soft topological space $(F_A, \tilde{\tau})$ is soft semi-closed set.

Proof. The proof follows from the Definition 3.8 □

The following Example 3.10. Shows that the converse implication of Theorem 3.9. is not true.

Example 3.10. Consider the soft topological space of Example 3.2. Here F_2, F_3, F_5 and F_9 are soft semi-closed sets but not soft closed sets.

Theorem 3.11. F_C be soft semi-closed in a soft topological space $(F_A, \tilde{\tau})$ if and only if $int(F_E) \tilde{\subseteq} F_C \tilde{\subseteq} F_E$ for some soft closed set F_E.

Proof. F_C is soft semi-closed if and only if F_C^c is soft semi-open if and only if there is a soft open set F_D such that $F_D \tilde{\subseteq} F_C^c \tilde{\subseteq} cl(F_D)$, by Theorem 3.5 if and only if there is a soft open set F_D such that $(cl(F_D))^c \tilde{\subseteq} F_C \tilde{\subseteq} F_D^c$ if and only if there is a soft open set F_D such that $int(F_D^c) \tilde{\subseteq} F_C \tilde{\subseteq} F_D^c$ if and only if there is a soft closed set F_D such that $int(F_E) \tilde{\subseteq} F_C \tilde{\subseteq} F_E$ where $F_E = F_D^c$. □

Theorem 3.12. A soft subset F_B in a soft topological space $(F_A, \tilde{\tau})$ is soft semi-closed if and only if $int(cl(F_B)) \tilde{\subseteq} F_B$.

Proof. F_B is soft semi-closed if and only if F_B^c is soft semi-open if and only if $F_B^c \tilde{\subseteq} cl(int(F_B^c))$ if and only if $F_B^c \tilde{\subseteq} cl((cl(F_B))^c)$, by definition if and only if $F_B^c \tilde{\subseteq} (int((cl(F_B)))^c$, if and only if $int(cl(F_B)) \tilde{\subseteq} F_B$. This completes the proof. □

Theorem 3.13. Let $(F_A, \tilde{\tau})$ be a soft topological space and $\{(F_{B_\alpha}) : \alpha \in \Delta\}$ be a collection of soft semi-closed sets in $(F_A, \tilde{\tau})$. Then $\cap_{\alpha\in\Delta} F_{B_\alpha}$ is also a soft semi-closed set.

Proof. Let $\{(F_{B_\alpha}) : \alpha \in \Delta\}$ be a collection of soft semi-closed sets in $(F_A, \tilde{\tau})$. Then for each $\alpha \in \Delta$, we have a soft closed set F_{C_α} such that $int(F_{C_\alpha}) \tilde{\subseteq} F_{B_\alpha} \tilde{\subseteq} F_{C_\alpha}$. Then $int(\cap_{\alpha\in\Delta} F_{C_\alpha}) \tilde{\subseteq} \cap_{\alpha\in\Delta} int(F_{C_\alpha}) \tilde{\subseteq} \cap_{\alpha\in\Delta} F_{B_\alpha} \tilde{\subseteq} \cap_{\alpha\in\Delta} F_{C_\alpha}$. Because $\cap_{\alpha\in\Delta} F_{C_\alpha} = F_C$ is soft closed set by prop.2.15(iii), then $\cap_{\alpha\in\Delta} F_{B_\alpha}$ is soft semi-closed set. □

Definition 3.14. Let $(F_A, \tilde{\tau})$ be a soft topological space and let F_B be a soft set in F_A.
(i) The soft semi-interior of F_B is the soft set $\tilde{\cup} \{F_C : F_C$ is soft semi-open and $F_C \tilde{\subseteq} F_B\}$ and is denoted by $s\text{-}int(F_B)$.
(ii) The soft semi-closure of F_B is the soft set $\tilde{\cap} \{F_C : F_C$ is soft semi-closed and $F_B \tilde{\subseteq} F_C\}$ and is denoted by $s\text{-}cl(F_B)$.

Clearly, $s\text{-}cl(F_B)$ is the smallest soft semi-closed set containing F_B and $s\text{-}int(F_B)$ is the largest soft semi-open set contained in F_B. By Theorem 3.7 and 3.13, we have $s\text{-}int(F_B)$ is soft semi-open set and $s\text{-}cl(F_B)$ is soft semi-closed set.

Example 3.15. Let the soft topological set $(F_A, \tilde{\tau})$ and the soft set $F_B = \{(e_1, \{a, b\}), (e_2, \{a, b, c\})\}$ be the same as in Example 3.2, we get $s\text{-}int(F_B) = F_B$.

Example 3.16. Let the soft topological set $(F_A, \tilde{\tau})$ and the soft set $F_B = \{(e_1, \{b, c\}), (e_2, \{a\})\}$ be the same as in Example 3.2, we get $s\text{-}cl(F_B) = F_B$.

From the definitions of soft semi-interior and soft semi-closure, we have the following theorem.

Theorem 3.17. Let $(F_A, \tilde{\tau})$ be a soft topological space and F_B be a soft set in F_A. We have $int(F_B) \tilde{\subseteq} s\text{-}int(F_B) \tilde{\subseteq} F_B \tilde{\subseteq}$

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s-cl(F_B) $\tilde{\subseteq}$ cl(F_B).

Proof. By Theorem 3.3, Theorem 3.9 and Definition 3.14. □

Theorem 3.18. Let $(F_A, \tilde{\tau})$ be a soft topological space and F_B be a soft set in F_A. Then the following hold.

(i) $(\text{s-cl}(F_B))^c = \text{s-int}(F_B^c)$

(ii) $(\text{s-int}(F_B))^c = \text{s-cl}(F_B^c)$

Proof.

(i) $(\text{s-cl}(F_B))^c = (\tilde{\cap}\{F_C : F_C$ is soft semi-closed and $F_C \tilde{\subseteq} F_B\})^c$.

$= \tilde{\cup}\{F_C^c : F_C$ is soft semi-closed and $F_C \tilde{\subseteq} F_B\}$

$= \tilde{\cup}\{F_C^c : F_C^c$ is soft semi-open and $F_C^c \tilde{\subseteq} F_B^c\} = \text{s-int}(F_B^c)$.

(ii) $(\text{s-int}(F_B))^c = (\tilde{\cup}\{F_C : F_C$ is soft semi-open and $F_C \tilde{\subseteq} F_B\})^c$.

$= \tilde{\cap}\{F_C^c : F_C$ is soft semi-open and $F_C \tilde{\subseteq} F_B\}$.

$= \tilde{\cap}\{F_C^c : F_C^c$ is soft semi-closed and $F_C^c \tilde{\subseteq} F_B^c\} = \text{s-cl}(F_B^c)$. □

Theorem 3.19. Let $(F_A, \tilde{\tau})$ be a soft topological space and let F_B and F_C be soft sets in F_A. Then the following hold.

(i) s-cl(F_ϕ) = F_ϕ and s-cl(F_A) = F_A.

(ii) F_B is soft semi-closed set if and only if F_B=s-cl(F_B).

(iii) s-cl(s-cl(F_B))=s-cl(F_B).

(iv) $F_B \tilde{\subseteq} F_C$ implies s-cl(F_B) $\tilde{\subseteq}$ s-cl(F_C).

(v) s-cl($F_B \tilde{\cap} F_C$) $\tilde{\subseteq}$ s-cl(F_B) $\tilde{\cap}$ s-cl(F_C).

(vi) s-cl($F_B \tilde{\cup} F_C$) =s-cl(F_B) $\tilde{\cup}$ s-cl(F_C).

Proof. (i) is obvious.

(ii) If F_B is soft semi-closed set, then F_B is itself a soft semi-closed set in F_A which contains F_B. So s-cl(F_B) is the smallest soft semi-closed set containing F_B and F_B =s-cl(F_B). Conversely, suppose that F_B=s-cl(F_B). Since s-cl(F_B) is a soft semi-closed set, so F_B is soft semi-closed set.

(iii) Since s-cl(F_B) is a soft semi-closed set therefore by part (ii) we have s-cl(s-cl(F_B))=s-cl(F_B).

(iv) Suppose that $F_B \tilde{\subseteq} F_C$. Then every soft semi-closed super set of F_C will also contain F_B. This means every soft semi-closed super set of F_C is also a soft semi-closed super set of F_B. Hence the soft intersection of soft semi-closed super sets of F_B is contained in the soft intersection of soft semi-closed super sets of F_C. Thus s-cl(F_B) $\tilde{\subseteq}$s-cl(F_C).

(v) Since $F_B \tilde{\cap} F_C \tilde{\subseteq} F_B$ and $F_B \tilde{\cap} F_C \tilde{\subseteq} F_C$. and So by part (iv) s-cl($F_B \tilde{\cap} F_C$) $\tilde{\subseteq}$ s-cl(F_B) and s-cl($F_B \tilde{\cap} F_C$) $\tilde{\subseteq}$s-cl(F_C) .
Thus s-cl($F_B \tilde{\cap} F_C$) $\tilde{\subseteq}$ s-cl(F_B) $\tilde{\cap}$ s-cl(F_C).

(vi) Since $F_B \tilde{\subseteq} F_B \tilde{\cup} F_C$ and $F_C \tilde{\subseteq} F_B \tilde{\cup} F_C$. So by part (iv) $F_B \tilde{\subseteq} F_C$ implies s-cl(F_B) $\tilde{\subseteq}$ s-cl(F_C). Then s-cl(F_B) $\tilde{\subseteq}$ s-cl($F_B \tilde{\cup} F_C$) and s-cl(F_C) $\tilde{\subseteq}$s-cl($F_B \tilde{\cup} F_C$), which is implies s-cl(F_B) $\tilde{\cup}$ s-cl(F_C) $\tilde{\subseteq}$s-cl($F_B \tilde{\cup} F_C$). Now, s- cl(F_B) , s-cl(F_C) is belong to soft semi-closed set in F_A which is implies that s-cl(F_B) $\tilde{\cup}$ s-cl(F_C) is belong to soft semi-closed set in F_A. Then $F_B \tilde{\subseteq}$s-cl(F_B) and $F_C \tilde{\subseteq}$ s-cl(F_C) imply $F_B \tilde{\cup} F_C \tilde{\subseteq}$ s-cl(F_B) $\tilde{\cup}$ s-cl(F_C). That is s-cl(F_B) $\tilde{\cup}$ s-cl(F_C) is a soft semi-closed set containing $F_B \tilde{\cup} F_C$. Hence s-cl($F_B \tilde{\cup} F_C$) $\tilde{\subseteq}$ s-cl(F_B) $\tilde{\cup}$ s-cl(F_C).
So, s-cl($F_B \tilde{\cup} F_C$)=s-cl(F_B) $\tilde{\cup}$ s-cl(F_C). □

Theorem 3.20. Let $(F_A, \tilde{\tau})$ be a soft topological space and let F_B and F_C be soft sets in F_A . Then the following hold.

(i) s-int(F_ϕ) = F_ϕ and s-int(F_A) = F_A.

(ii) F_B is soft semi-open set if and only if F_B = s-int(F_B).

(iii) s-int(s-int(F_B))=s-int(F_B).

(iv) $F_B \tilde{\subseteq} F_C$ implies s-int(F_B) $\tilde{\subseteq}$ s-int(F_C).

(v) s-int(F_B)$\tilde{\cap}$s-int(F_C) $\tilde{\subseteq}$s-int($F_B \tilde{\cap} F_C$).

(vi) s-int($F_B \tilde{\cup} F_C$) =s-int(F_B) $\tilde{\cup}$ s-int(F_C).

Proof. (i) is obvious.

(ii) If F_B is soft semi-open set, then F_B is itself a soft semi-open set in F_A which contains F_B. So s-int(F_B) is the largest soft semi-open set contained in F_B and F_B=s-int(F_B). Conversely, suppose that F_B=s-int(F_B) . Since s-int(F_B) is a soft semi-open set, so F_B is soft semi-open set in F_A.

(iii) Since s-int(F_B) is a soft semi-open set therefore by part (ii) we have s-int(s-int(F_B))=s-int(F_B).

(iv) Suppose that $F_B \tilde{\subseteq} F_C$. Since s-int(F_B) $\tilde{\subseteq} F_B \tilde{\subseteq} F_C$. s-int(F_B) is a soft semi-open subset of F_C, so by definition of s-int(F_C) , s-int(F_B) $\tilde{\subseteq}$ s-int(F_C).

(v) Since $F_B \tilde{\subseteq} F_B \tilde{\cap} F_C$ and $F_C \tilde{\subseteq} F_B \tilde{\cap} F_C$. and So by part (iv) s-int(F_B) $\tilde{\subseteq}$ s- int($F_B \tilde{\cap} F_C$) and s-int(F_C) $\tilde{\subseteq}$s-int($F_B \tilde{\cap} F_C$). So that s-int(F_B) $\tilde{\cap}$ s-int(F_C) $\tilde{\subseteq}$ s-int($F_B \tilde{\cap} F_C$), since s-int($F_B \tilde{\cap} F_C$) is a soft semi-open set.

(vi) Since $F_B \tilde{\subseteq} F_B \tilde{\cup} F_C$ and $F_C \tilde{\subseteq} F_B \tilde{\cup} F_C$ and So by part (iv) $F_B \tilde{\subseteq} F_C$ implies s-int(F_B) $\tilde{\subseteq}$ s-int(F_C). Then s-int(F_B) $\tilde{\subseteq}$ s-int($F_B \tilde{\cup} F_C$) and s-int(F_C) $\tilde{\subseteq}$s-int($F_B \tilde{\cup} F_C$), which is implies s-int(F_B) $\tilde{\cup}$ s-int(F_C) $\tilde{\subseteq}$ s-int($F_B \tilde{\cup} F_C$). Now, s-int(F_B) , s-int(F_C) is belong to soft semi-open set in F_A which is implies that s-int(F_B)$\tilde{\cup}$ s-int(F_C) is belong to soft semi-open set in F_A. Then $F_B \tilde{\subseteq}$ s-int(F_B) and $F_C \tilde{\subseteq}$ s-int(F_C) imply $F_B \tilde{\cup} F_C \tilde{\subseteq}$ s-int(F_B)$\tilde{\cup}$ s-int(F_C). That is s-int(F_B) $\tilde{\cup}$ s-int(F_C) is a soft semi-open set containing $F_B \tilde{\cup} F_C$. Hence s-int($F_B \tilde{\cup} F_C$) $\tilde{\subseteq}$ s-int(F_B) $\tilde{\cup}$ s-int(F_C). So, s-int($F_B \tilde{\cup} F_C$)=s-int(F_B) $\tilde{\cup}$ s- int(F_C). □

Theorem 3.21. Let $(F_A, \tilde{\tau})$ be a soft topological space and let F_B be soft sets in F_A. Then the following hold.

(i) s-cl(cl(F_B))=cl(s-cl(F_B))=cl(F_B)

(ii) s-int(int(F_B))=int(s-int(F_B))=int(F_B)

Proof. (i) Let $(F_A, \tilde{\tau})$ be a soft topological space and because int(F_B) is soft open set, we have int(F_B) is soft semi-open set by Theorem 3.3. So we can get s-int(int(F_B))=int(F_B) by Theorem 3.19(ii). By Theorem 3.17, we have int(F_B) $\tilde{\subseteq}$ s-int(F_B) $\tilde{\subseteq} F_B$, then we can get int(F_B) $\tilde{\subseteq}$ int(s- int(F_B)) $\tilde{\subseteq}$ int(F_B) and so int(s-int(F_B))=int(F_B). This completes the proof.

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(ii) Because $cl(F_B)$ is soft closed set, we have $cl(F_B)$ is soft semi closed set by Theorem 3.9. So we can get s-int(int(F_B))=int(F_B) by Theorem 3.18(ii). By Theorem 3.17, we have $F_B \tilde{\subseteq}$ s-cl(F_B) $\tilde{\subseteq}$ cl(F_B), then we can get cl(F_B) $\tilde{\subseteq}$ cl(s-cl(F_B)) $\tilde{\subseteq}$ cl(F_B) and so cl(s-cl(F_B))=cl(F_B). This completes the proof. □

Theorem 3.22. Let $(F_A, \tilde{\tau})$ be a soft topological space and let F_B be soft sets in F_A. Then the following are equivalent.

(i) F_B is soft semi closed set.
(ii) int(cl(F_B)) $\tilde{\subseteq}$ F_B .
(iii) cl(int(F_B^c)) $\tilde{\supseteq}$ F_B^c
(iv) F_B^c is soft semi-open set.

Proof. (i) ⇒ (ii) If F_B is soft semi-closed set, then there exist soft closed set F_C such that int(F_C) $\tilde{\subseteq}$ F_B $\tilde{\subseteq}$ F_C⇒int(F_C) $\tilde{\subseteq}$ F_B $\tilde{\subseteq}$cl(F_B) $\tilde{\subseteq}$ F_C . By the property of interior, we have int(cl(F_B)) $\tilde{\subseteq}$ int(F_C) $\tilde{\subseteq}$ F_B .

(ii)⇒(iii) int(cl(F_B)) $\tilde{\subseteq}$ F_B⇒F_B^c $\tilde{\subseteq}$ int(cl(F_B))^c =cl(int(F_B^c)) $\tilde{\supseteq}$ F_B^c.

(iii)⇒(iv) F_C=int(F_B^c) is an soft open set such that int(F_B^c) $\tilde{\subseteq}$$F_B^c$ $\tilde{\subseteq}$ cl(int(F_B^c)) , hence F_B^c is soft semi-open set.

(iv)⇒(i) As F_B^c is soft semi-open set, there exists an soft open set F_C such that $F_C$$\tilde{\subseteq}$$F_B^c$ $\tilde{\subseteq}$ cl(F_C) ⇒F_C^c is a soft closed set such that $F_B$$\tilde{\subseteq}$$F_C^c$ and $F_B^c$$\tilde{\subseteq}$cl($F_C$)⇒int($F_C^c$) $\tilde{\subseteq}$$F_B$. Hence F_B is soft semi-closed set. □

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